

The 2D Tyre Modelling conundrum and how to fix it using setup referencing

Over the last couple of months I have been involved in a number of different car modelling projects. These have ranged as categories as diverse as stock cars, sports cars to open wheelers. Yet despite the diversity I have been seeing the same themes emerging. Principally when using a 2D tyre model (Traction circle radius as a function of load only) you have great correlation but the sensitivity of the model to very fine changes is extremely small. The trends still point you in the right direction so it's still useful but the changes are much smaller than what you would see on an actual car. The purpose of this article is to explore why this is, and propose a simple fix for it that I will term setup referencing.

Let me add from the outset that this is an article that is designed to get you thinking. What I'm proposing here is not the final picture of how to model a tyre. Rather consider this a discussion paper to get you all thinking about the mechanisms that drive what we see when we make a setup change on a car.

To kick things off let's explore what your getting when you refine a 2D tyre model. The crux of the 2D tyre model is effectively to determining the traction circle radius (or Pacjeka D term) as a function of load only. This can be done either from tyre test rig data or using the ChassisSim tyre force modelling toolbox to get the tyre model. When you have done your job right you should be looking at correlation that looks like this,

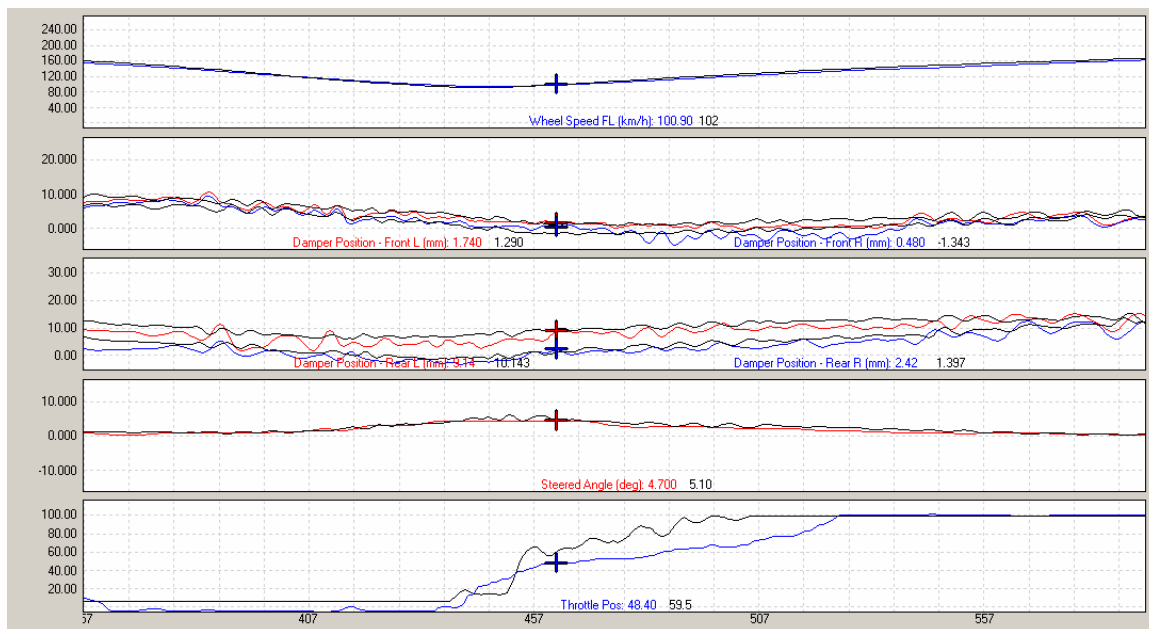


Fig-1 – Typical 2D tyre modelling correlation using ChassisSim.

On a number of different articles I have commented that when you create a 2D tyre model you are taking a snapshot of the tyre at a particular pressure and internal temperature condition. This is illustrated in Fig-2

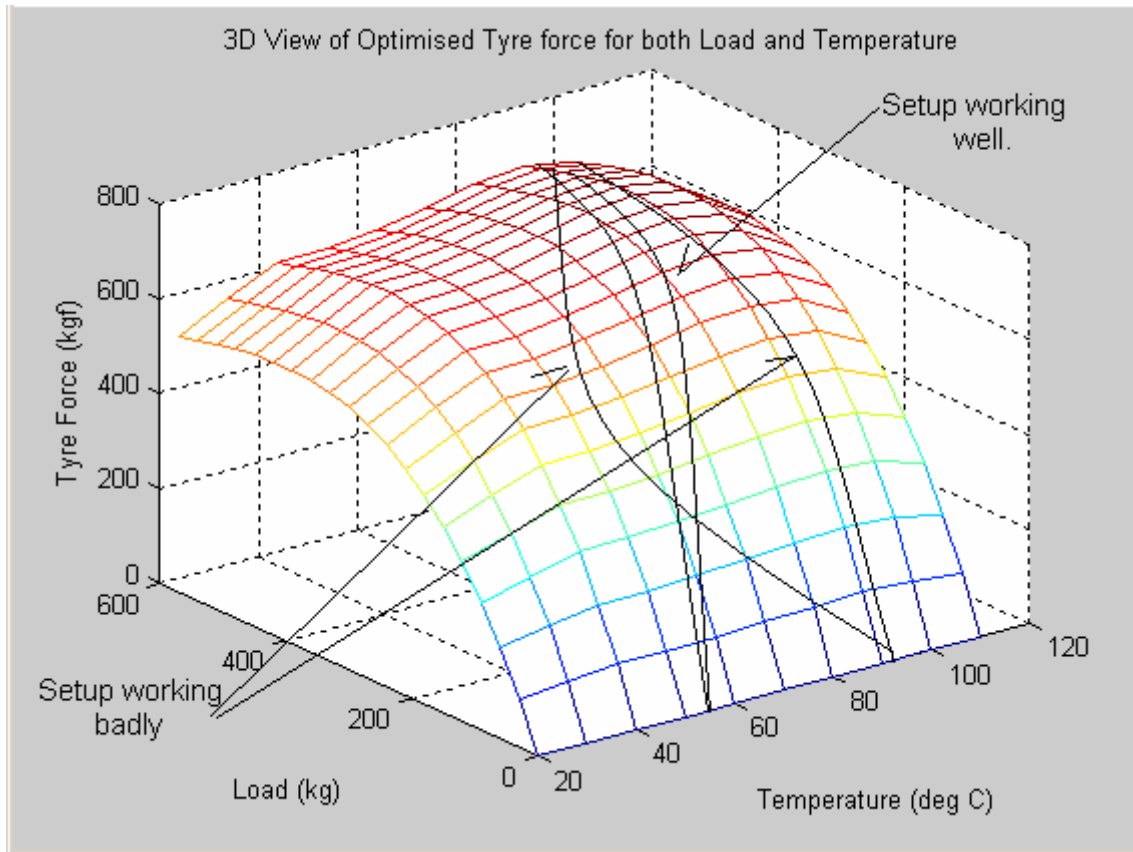


Fig-2 – An illustration of what you get from a 2D tyre model.

The setup working well line is the bit of the tyre you have typically taken the snapshot off. That being said most people would look at this and say great we are ready to start simulating.

However where 2D models struggle is sensitivity. Let's illustrate this with some hard numbers that I'm going to take from a V8 Supercar. In reality I could illustrate this with any car, but I'm using a Supercar because the change we are about to discuss is well known and is a very good case in point.

To frame this discussion let's illustrate some typical V8 Supercar numbers. Typical setup values in Table – 1,

Table-1 – Typical V8 Supercar setup

| Parameter | Value |
|-------------------|---------|
| Front roll centre | 100 mm |
| Rear roll centre | 240 mm |
| Front Wheel rate | 60 N/mm |

| | |
|------------------------|-----------|
| Rear Wheel rate | 60 N/mm |
| Front bar wheel rate | 40 N/mm |
| Rear bar wheel rate | 10 N/mm |
| Front track/Rear track | 1.6m/1.6m |
| Front tyre spring rate | 305 N/mm |
| Rear tyre spring rate | 305 N/mm |
| c.g height | 0.45m |

A representative 2D tyre model for a V8 supercar is shown in Table – 2.

Table – 2 – Representative 2D tyre model for a V8 Supercar.

| Parameter | Value |
|----------------------------------|--------|
| Initial co-efficient of friction | 2.2 |
| Peak Load | 850 kg |

Now that we have this information to hand let's explore some setup sensitivity parameters. One big change in a V8 Supercar is a rear roll centre change. Typically a 10mm will produce a measurable change. To keep things simple let's apply a lateral acceleration of 1.4g and use this acceleration to estimate tyre loads and approximate cornering force. The results are shown in Table – 3,

Table – 3 – Predictive numbers for a Rear roll centre change.

| Setup | Load FL | Load FR | Load RL | Load RR | FyR | V_pred |
|---------|---------|---------|---------|---------|--------|--------|
| RRC 240 | 674.96 | 133.82 | 694.58 | 54.47 | 9993.5 | 81.47 |
| RRC 250 | 671.61 | 137.16 | 697.93 | 51.1 | 9938 | 81.27 |

All Loads are shown in kg, the lateral forces are shown in N and V_pred is in km/h.

What has been presented in Table – 3 is a very simplified analysis. We are simply taking a static weight and applying a load transfer to it for a typical low speed corner. However what is revealing in Table-3 is that while the speeds are representative the magnitude of the change is very small. With the 10mm rear roll centre change, the tyre loads have changed by a mere 3 kg, and the speed has changed by 0.2 km/h. Yet in practice on the car you'll typically see a change of 0.4 – 1km/h. In reality the truth is somewhere in the middle but this illustrates that a 2D model gives you a very good broad brush stroke but does lack fidelity.

The reason the change is so small is that the delta forces produced by the 2D model are miniscule. This is primarily due to the value of the peak load of 850 kg. When we crunch the numbers the magnitudes of the changes are so small because the gradients of the tyre force curve are tiny. This is due to the outside tyre loads approaching the optimum tyre load of the model. Don't take my word for it, do the numbers yourself. Remember a simple 2D tyre model is given by,

$$F_y = k_a (1 - k_b \cdot F_z) \cdot F_z$$
$$L_p = \frac{1}{2 \cdot k_b} \quad (1)$$

where

- k_a = initial coefficient of friction
- k_b = drop off of coefficient with load
- F_z = load on the tyre

There are a number of things you can do to mitigate this but ultimately it compromises your model in other areas. For example to go for setup sensitivity we can drop the peak load. It gives you sensitivity, but it compromises your accuracy for high speed corners and adding more downforce. Clearly there is something else at play here.

To further ram home this point let's consider the nature of load transfer for an independent suspension. I spoke this at length in my article on V8 Supercars which I entitled a Brave New World. Remember the highlights for an independent suspension. Due to the nature of force application points, the suspension geometry will effectively pin the outside tyre, magnifying the load transfer on this tyre. This load transfer is not equal and opposite like it was on the live axle. I can tell you right now this is where the 2D tyre model will run out of steam. Let's crunch the numbers to illustrate this. To keep our discussion simple we'll use the tyre model and I'll add 15 kg to the outside front tyre. The results are illustrated in Table – 5

Table-5 – Load Transfer numbers for a front suspension

| Setup | Load FL | Load FR | Fyf | V_pred |
|----------|---------|---------|-------|--------|
| FRC std | 674.96 | 133.82 | 11432 | 84.02 |
| FRC proj | 690 | 130.0 | 11426 | 84.0 |

This would be the case of a front rear roll centre change of 20mm. As we can see rather than losing cornering speeds it remains level. This is quite different from what you would expect, but what this illustrates is the limitations of using a 2D tyre model.

At this point it would be wise for me to not some historical observations about using a 2D tyre model. It has been my experience that while a 2D tyre model does lack sensitivity ultimately though it will point you in the right direction for most mechanical setup changes. However bare in mind the changes you are looking for are extremely small. This is due to the nature of the tyre model and the way the simulator dives the car. If you are content with this a 2D model is a fantastic tool. However if you want to quantify everything we need to use something else.

What we can divine from all this is to be very careful when someone claims a 2D model can give you great model sensitivity. All the numbers we have crunched in Table- 1 -5 using

equation 1 will tell you, that you will be very hard pressed to achieve model sensitivity. If you are getting model sensitivity there are a number of factors at play here,

- They are using a non linear table for Traction circle radius vs load.
- Camber effects are coming into play.
- Or something isn't quite right.

Either way you don't have to be a rocket scientist to figure out we are going to need something else to help us in aiding model sensitivity.

To figure out a way out of this mess let's consider what the tyre is actually doing. We'll kick off this discussion by illustrating a tyre schematic in Fig-3,

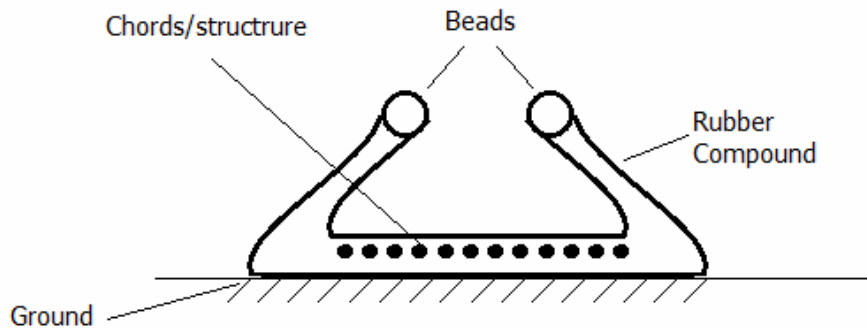


Fig-3 – Tyre schematic

What happens is as we apply different setup conditions to the tyre the internal temperature and hence the tyre pressure changes. At this point you might be thinking so what? There are a number of factors that come into play here. The first is the effect this has on the contact patch variation.

$$A_{c.p} = \frac{F_z}{P_T} \quad (2)$$
$$l_{c.p} = \frac{A_{c.p}}{w_t}$$

where,

- $A_{c.p}$ = contact patch area (m²)
- F_z = vertical load on the tyre (N).
- w_t = width of the tread (m).
- l_{cp} = length of the contact patch (m)

As we can see as the Pressure goes up the contact patch length goes down. Consequently there is less area in the contact patch. So consequently there is less area for the forces to be applied and the tyre force will go down. In the converse situation when the temperature and pressure is not optimal, there is more contact patch length and hence greater area. But the core temperature of the tyre isn't up to temperature yet. This is where the second effect comes into play. From the research that Michelin did in constructing the Michelin TaMe Tire model the overall friction of the tyre is given by,

$$\mu = fn(P_L, V_s, T_{INT}) \quad (3)$$

Where,

μ = Co-efficient of friction of the tyre.

P_L = Local tyre pressure applied at the tyre

V_s = Sliding velocity of the tyre.

T_{INT} = Core temperature of the tyre

This combined with the contact pressure will have a significant effect on the tyre forces. This is what we are approximating with setup referencing.

So reviewing this let me propose a modification to the traction circle radius equation. The modification I'm going to propose is the following,

$$TC_{RAD} = fn(F_z)fn(P_t) \quad (4)$$

What we have here is the 2D model that we discussed in equation (1) multiplied by a multiplication factor that is a function of hot tyre pressure. This function will look something like this,

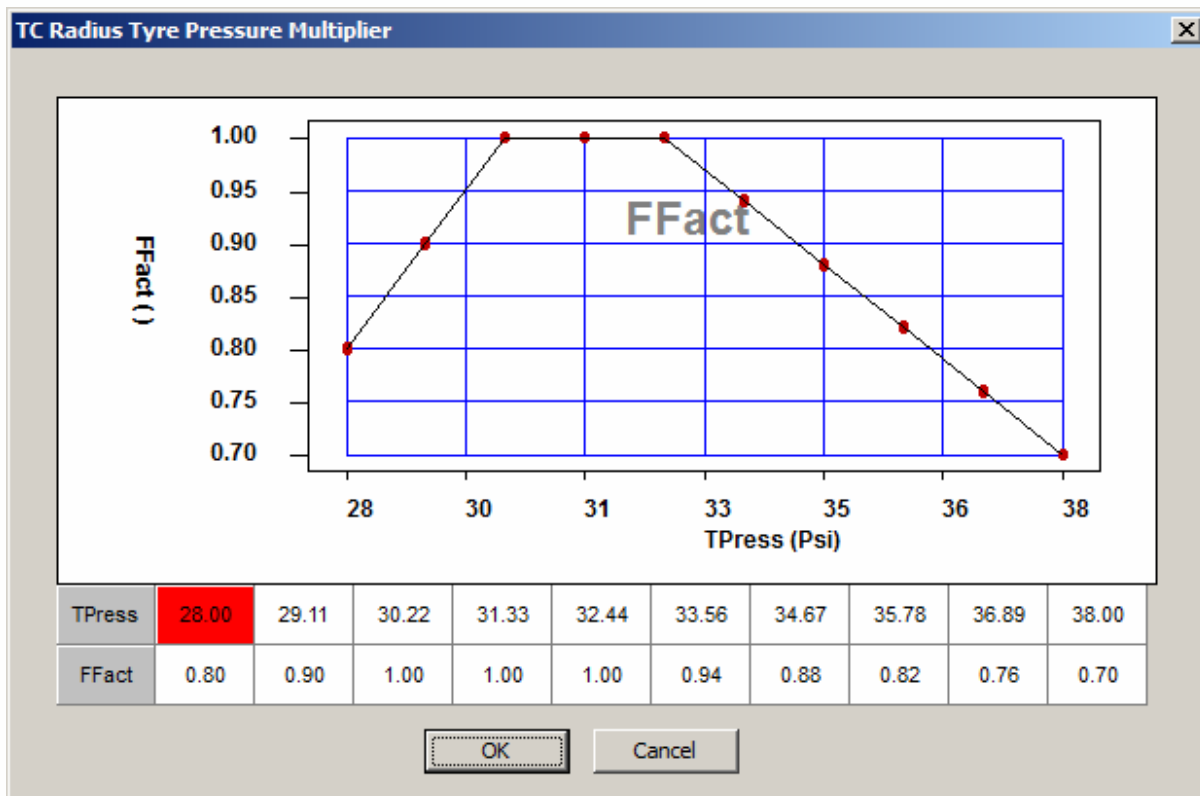


Fig-4 – FFact multiplier of Traction Circle radius vs Tyre Pressure.

At this point some of you might be thinking what drugs am I on and I think a fair few of you might be a tad annoyed but let's take a moment to review a things. From Fig-1 we know we can achieve very good correlation using a 2D model. However when we investigate the nature of the numbers, the sensitivity needs to be improved. Furthermore if we are not careful it can lead us up the garden path. Also any body that has spent more than 5 minutes with a real race car knows how critical it is to get the tyre pressures right. Consequently there must be something else coming into play and all the results from the Michelin TaMe Tire model would indicate very strongly that tyre pressure plays a critical role. You also can confirm this from running the car.

To quantify this we are going to have to run a series of tests on the car. The test matrix we will run and what to record is recorded in Table – 1

Table – 6: Tests to run for Setup referencing

| Test No | Setup | What to record |
|---------|-----------------------------|-----------------------|
| 1 | Base Setup | Tyre Pressures + data |
| 2 | Base Setup + Fnt TP -2p.s.i | Tyre Pressures + data |
| 3 | Base Setup + Fnt TP -1p.s.i | Tyre Pressures + data |
| | Base Setup + Fnt TP +1p.s.i | Tyre Pressures + data |

| | | |
|---|------------------------------|-----------------------|
| 5 | Base Setup + Fnt TP +2p.s.i | Tyre Pressures + data |
| 6 | Base Setup + Rear TP -2p.s.i | Tyre Pressures + data |
| 7 | Base Setup + Rear TP -1p.s.i | Tyre Pressures + data |
| 8 | Base Setup + Rear TP +1p.s.i | Tyre Pressures + data |
| 9 | Base Setup + Rear TP +2p.s.i | Tyre Pressures + data |

It is critical for every run you are recording the hot tyre pressures. You'll also note I've indicated a sweep of +/- 1 p.s.i. In reality this is going to be dictated by your running experience and what the car wants.

The critical thing to be paying attention to here is the lateral acceleration data. Due to the nature of this test if there is a change it's very easy to isolate what caused it. We can also easily quantify the change. For example if we are dealing with a front tyre pressure change the delta FFact will look like this,

$$\Delta FF_{FACT} = \frac{a_{y_cur} - a_{y_ref}}{wdf \cdot a_{y_ref}} \quad (5)$$

Here we have

- ΔFF_{FACT} = Change in Force factor
- a_{y_cur} = Measured lateral acceleration for the change.
- a_{y_ref} = Measured lateral acceleration for the base setup.
- wdf = Weight distribution at the front.

Clearly if you are measuring a change at the rear you simply measure substitute the wdf with weight distribution at the rear. Once this data is calculated, Fig-4 can be readily populated.

The next step in the process is to match our setup data with the tyre model. The critical thing to note here is that the more roll centre you apply or the stiffer you make one end of the car or the more static tyre pressure you apply, the hot core temperature and pressure of the tyre will increase. These adjustments are a race engineer's primary tools when they are dialling in core tyre temps and pressures. What we are going to do here is match this to our baseline setup and attach multipliers to our baseline setup. This is the key to setup referencing. While this is not perfect at least it gets us into the ball park and this is our goal.

The most effective way to do this is to use your race car simulator to match your expectations with what occurred with the car. Some of you think this is fudging. It isn't. What you are doing is using the race car simulator to quantify what's going on with the car. This to me is the biggest benefit of using race car simulation because it forces you to understand what is going on with the car. This is the biggest pay off to using race car simulators which is why I maintain you must do it yourself.

So to achieve this we need to do assign the base setup and the pressure multipliers. As an example let me show you the sample controls in ChassisSim,

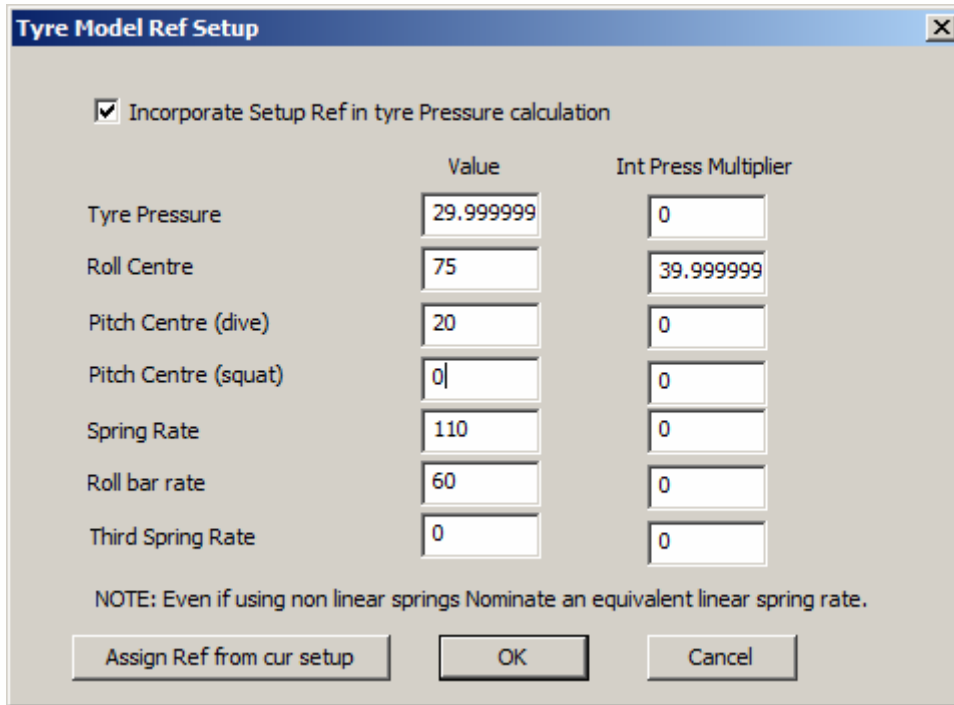


Fig-5 – Setup reference dialog from ChassisSim.

As can be seen the base setup is indicated in the first column. Please note The tyre pressure you enter here is the warm tyre pressure from the base setup. This is very important. The next step in the process is to assign the pressure multipliers. Some baseline suggestions for numbers in p.s.i is illustrated below,

Table-7 – Some rough rules of thumb for the setup sensitivity parameters

| Parameter | Value |
|---------------------------|---|
| Tyre pressure | $1 * (TP_{cur} - TP_{ref}) / TP_{ref}$ |
| Roll centre | $30-40 * (rc_{cur} - rc_{ref}) / h$ |
| Pitch Centre | $30-40 * (pc_{cur} - pc_{ref}) / h$ |
| Spring/ Third Spring Rate | $10 * (k_{cur} - k_{ref}) / k_{ref_total}$ |
| Bar rate | $10 * (k_{cur} - k_{ref}) / k_{ref_total}$ |

The variables here are,

- TP_{cur} = Current tyre pressure
- TP_{ref} = Tyre pressure of the reference setup.
- rc_{cur} = Current roll centre
- rc_{ref} = Roll centre of the reference setup.
- pc_{cur} = Current pitch centre
- pc_{ref} = Pitch centre of the reference setup.
- h = c.g height.
- k_{cur} = Current spring, third spring or bar rate.

k_{ref} = Spring, third spring or bar rate of the reference setup.

k_{ref_total} = Sum of all the spring and bar rates of the reference setup.

These numbers are subject to car factors such as motion ratios and parameters. However the key to getting this right is to use these pressure multipliers to dial in some typical changes in setup and dial this. Remember at this point we are using the race car simulator as the ultimate motorsport calculator.

Also too bear in mind what we are presenting here is an approximation and don't forget that. Before a lot of the readership turn their noses in disgust at this let me just remind you of some approximations that work pretty well,

- In most CFD analysis for aircraft air viscosity is ignored. However it gets you in the ball park.
- The calculation of damping ratios ignores the tyre spring rate. It still doesn't stop it being a valuable tool.

Remember the key here is we are using setup referencing to quantify what's going on with the car. From that we can make the appropriate determination of setup changes.

In closing then setup referencing provides a very interesting perspective about how we can use race car simulation to classify the sensitivity of setup changes with the car. Also too just remember this is not a technique etched in stone. Rather this is something to get you thinking. Consider this as a first step into quantifying just how sensitive a setup change truly is. Remember the key to this technique is to classify the tyre force sensitivity to hot pressure changes and matching that to our setup information. Consequently this is a useful approximation for extending the 2D model of the Traction circle radius vs load characteristic of the tyre. This knowledge provides valuable insights to determine how to make your race car go faster.